Zbl 483.51010

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On the approximation of convex, closed plane curves by multifocal ellipses. (In English)

Essays in statistical science, Pap. in Honour of P.A.P. Moran, J. Appl. Probab., Spec. Vol. 19A, 89-96 (1982).

[For the entire collection see Zbl 477.00023.]

A multifocal ellipse is the set of all points P which satisfy the relation

$$\sum_{i=1}^{n} PF_i = C > \min_{(Q)} \sum_{i=1}^{n} QF_i,$$

where juxtiposition denotes distance. In 1937, E. Weisfeld (Vazsonyi) asked if a convex curve could be approximated arbitrarily by multifocal ellipses. The authors gave a negative answer to that question in q958, by showing an equilateral triangle cannot be approximated arbitrarily by such ellipses. This paper extens that result. The authors show here that the limiting curve of a certain type of family of multifocal ellipses with bounded foci can have at most one line segment. They show further that when all the foci of a certain family of multifocal elipses tend to ∞, then the limiting figure is an infinite line or an ellipse. Certain "distance integrals" are seen to be approximable by multifocal ellipses. Whether or not these are the only curves which can be arbitrarily approximated by multifocal ellipses remains an open problem. It should be noted that there is a misprint in the second equation after (1.1); then F_2 should be P_2. Also, in the binomial expansion, p.93, a 2 is missing in the denominator of the third term. This does not effect what follows, since in the application $r_{it} \to \infty$ and $2r_{it}$ does also and vice versa.

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Classification:
51K10 Synthetic differential geometry
51M25 Length, area and volume (geometry)

Keywords:
approximation; convex curves; multifocal ellipse