Zbl 483.05053

Chung, F.R.K.; Erdős, Paul; Graham, Ronald L.

On the product of the point and line covering numbers of a graph. (In English) Combinatorial mathematics, 2nd int. Conf., New York 1978, Ann. New York Acad. Sci. 319, 597-602 (1979).

[For the entire collection see Zbl 468.00005.]

For a graph G = (V, E) let the point covering number  $\alpha_0(G)$  and the line covering number  $\alpha_1(G)$  be defined as follows:

 $\alpha_0(G) = \min\{|X| : X \subseteq V \text{ and every } e \in E \text{ contains some } x \in X\}$   $\alpha_1(G) =$  $\min\{|Y|:Y\subseteq E \text{ and every } v\in V \text{ is contains in some } y\in Y\}.$  The authors answer conjectures of F. Harary and J.A. Kabell (in the same proceedings, Zbl 483.05037) by proving that:

$$\alpha_0(G)\alpha_1(G) \ge n-1,$$

and

$$\alpha_0(G)\alpha_1(G) \le \begin{cases} \left\{ \frac{n^2-1}{2} \text{ for n odd,} \right. \\ \left. \frac{n^2-4}{2} \right. \end{cases}$$
 for n even.

(cases of equality are characterized).

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Classification:

05C70 Factorization, etc.

05C35 Extremal problems (graph theory)

point covering number; line covering number