Zbl 469.10036

Erdős, Paul

Some extremal problems on divisibility properties of sequences of integers. (In English)

Fibonacci Q. 19, 208-213 (1981). [0015-0517]

The author studies various properties of subsets $A = \{a_1 < a_2 < \cdots < a_k \leq n\}$ of integers from 1 to n. The sequence A is said to have the property $P_r(n)$ if no a_i divides the product of r of the others. A is said to have the property $P(n)$ if no a_i divides the product of the others. It is said to have the property $Q(n)$ if the products $a_i a_j$ are all distinct. To his results in [Izv. Nauk. Inst. Mat. Mekh. Univ. Tomsk 2, 74-82 (1938; Zbl 020.00504)] he adds further results in this paper. to state them he defines some arithmetical functions. Let S_n denote all the integers from 1 to n. Let $f_r(n)$ denote the smallest integer such that S_n can be decomposed into $g(n)$ sets each with the property $Q(n)$. The author proves the results $2n^{1/2} > f_r(n) \gg n^{1/2}/\log n$ and $2n^{1/2} > g(n) \gg n^{1/3}/\log n$. Next he proves the results that for every $\varepsilon > 0$, there holds $n^{1-1/r} \gg f_r(n) \gg \varepsilon^{n^{1-(1/r)-\varepsilon}}$. Another result about the divisor properties is that if A has the property that the product of any two a_i is a multiple of all the others, then $\log(\max k)$ is asymptotic to $\left(\frac{2\log 2}{3}\right)\frac{\log n}{\log\log n}$ as $n \to \infty$. I wish to quote on more result. Let $F(n)$ be the smallest integer for which S_n can be decomposed into $F(n)$ sets $\{A_i\} 1 \leq i \leq F(n)$ each having the property P. Then

$$F(n) = n\mathrm{Exp}((-c + o(1))(\log n \log\log n)^{1/2}).$$

The author discusses many other deep results about sets A with $a_i + a_j$ all distinct and so on. As usual the paper is full of interesting problems and partial solutions (sometimes complete solutions) and i would request the readers to consult the paper for more details.

K.Ramachandra

Classification:
11B83 Special sequences of integers and polynomials
11B05 Topology etc. of sets of numbers
11B13 Additive bases

Keywords:
extremal problems; divisibility properties; sequences of integers