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An extremal problem in generalized Ramsey theory. (In English)

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Chvátal proved that the Ramsey number $R(K_m, T)$ of a complete graph on m vertices and a tree on n vertices is given by $(m-1)(n-1)+1$. In this article, the authors consider connected graphs on n vertices and q edges satisfying the equation $R(K_m, G) = (m-n)(n-1)+1$. Such graphs are called m- good. They then define $f(m,n)$ as the largest q for which every connected graph on n vertices and q edges is m-good. Similarly, $g(m,n)$ is the largest q for which there exist a connected m-good graph on n vertices with q edges. The authors then establish the following bounds: 1. For all $n \geq 4, f(3,n) \geq (17n+1)/15$, 2. For every $\varepsilon > 0$, if n is sufficiently large, then $f(3,n) < (27/4+\varepsilon)n(\log n)^2$, 3. There exist positive constants A, B so that: $An^{3/2}(\log n)^{1/2} < g(3,n) < Bn^{5/3}(\log n)^{2/3}$ for all n sufficiently large. the authors also pressent vounds for $f(m,n)$ and $g(m,n)$ for arbitrary m and pose the question of determining whether $f(n)/n \to \infty$ as $n \to \infty$.

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