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On bases with an exact order. (In English)

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The statement of the results in this paper require the following definitions: Let A be a set of non-negative integers. A is said to be an asymptotic basis of order r (written $\mathrm{ord}(A) = r$) if r is the least integer such that every sufficiently large integer is expressible as the sum of at most r integers from A (allowing repetition). Also, A is said to have exact order s (written $\mathrm{ord}^*(A) = s$) if s is the least integer for which this is possible with exactly s integers from A. There are basis not having exact order. The following results are established:
I. A basis $A = \{a_1, a_2, \ldots\}$ has an exact order if and only if

$$\gcd\{a_{k+1} - a_k | k = 1, 2, \ldots\} = 1. \quad (1)$$

II. Let $g(r) = \max\{\mathrm{ord}^*(A)\}$ subject to (1), and $\mathrm{ord}(A) = r$. Then,

$$\frac{1}{4}(1 + 0(1))r^2 \le g(r) \le \frac{5}{4}(1 + 0(1))r^2.$$

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11B13 Additive bases

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