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Erdős, Paul; Hall, R.R.

The propinquity of divisors. (In English)

Bull. Lond. Math. Soc. 11, 304-307 (1979). [0024-6093]

In [J. London Math. Soc. 39, 692-696 (1964; Zbl 125.08602)], *P.Erdős* stated that, if $\beta > \log 3 - 1$, then the sequence of integers n with two divisors d,d' satisfying $d < d' < d(1-(\log n)^{-\beta})$ has asymptotic density 0. An improvement of this by the present authors shows in effect that one can replace the term $(\log n)^{-\beta}$ in the above by $3^{-\lambda(n)}(\log d)^{1-\log 3}$ where

$$\lambda(n) = (1-\varepsilon)\sqrt{\{2\log\log n \cdot \log\log\log\log n\}}$$

and $\varepsilon > 0$ is fixed. The proof (of an equivalent form of this result) depends on estimating certain weighted sums over the integers n¡ x of the required type; the case when $d > x^{\delta}(\delta > 0)$ is easier to handle, and indeed a slightly stronger result is derived here.

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Classification:
11N05 Distribution of primes
11N37 Asymptotic results on arithmetic functions
11K65 Arithmetic functions (probabilistic number theory)

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