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Colorful partitions of cardinal numbers. (In English)

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Let $\varkappa$, λ, μ, ν be infinite cardinal numbers. Let $[\varkappa]^2$ denote the set of all two element subsets of $\varkappa$, and consider $[\varkappa]^2$ as the set of edges for the complete graph on $\varkappa$ vertices. The authors define the relation $CP(\varkappa,\mu,\nu)$ to hold if there is an edge colouring R: $[\varkappa]^2 \to \mu$ with μ colours such that for every proper ν size subset X of $\varkappa$ there is a vertex x in $\varkappa - X$ such that the edges between x and the vertices in X receive at least $\min(\mu\nu)$ colours. The relation $CP^\sharp(\varkappa,\mu,\nu)$ holds if there is such a colouring which is one-to-one on the edges between x and the vertices in X. There are related properties BP and $BP^\sharp$, where $BP(\varkappa,\lambda,\mu,\nu)$ holds if there is a colouring R: $\varkappa\times\lambda\to\mu$ of the complete $\varkappa$, λ bipartite graph with μ colours, such that for every ν size subset X of $\varkappa$ there is a point x in λ such that the edges between x and the vertices in X receive at least $\min(\mu,\nu)$ colours. The paper is devoted to a discussion of the properties BP and $BP^\sharp$. From these, properties of CP and $CP^\sharp$ are deduced, sufficient to characterize completely CP and $CP^\sharp$ under the assumption of the generalized continuum hypothesis.

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04A20 Combinatorial set theory
04A10 Ordinal and cardinal numbers; generalizations
04A30 Continuum hypothesis and generalizations
05C15 Chromatic theory of graphs and maps

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