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Set theoretic, measure theoretic, combinatorial, and number theoretic problems concerning point sets in Euclidean space. (In English)

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The author states several combinatorial statements putting in contrast the corresponding situations for the finite and the infinite case respectively and that in general the case of finite sets is more complicate than the one of transfinite sets. E.g. if S is any infinite set in E_k, then S contains an equinumerous set S_1 such that all distances between points of S_1 are distinct (cf. *P.Erdős*, Proc. Am. Math. Soc. 1, 127-141 (1950; Zbl 039.04902). On the other hand, let $f_k(n)$ be the largest integer such that any n-point-set S in E_k contains an $f_k(n)$-subset S_1 such that all distances of points of S_1 are distinct. "The exact determination of $f_k(n)$ seems hopoless...". A plausible conjecture is $f_1(n) = (1+o(1))n^{1/2}$, where $g_1(n) = \max k$ such that any strictly increasing k-sequence $a_1 < a_2 < \cdots < a_k$ of natural numbers $\leq n$ has all distinct differences $a_j - a_i$. The author conjectures that $g_1(n) = n^{1/2} + 0(1)$. He conjectures that $f_1(n) = g_1(n) = n^{1/2} + 0(1)$. Let n_k be the smallest integer such that $f_k(n_k) = 3$; e.g. $n_2 = 9$; it is not known whether $n_k^{1/k} \to 1$. Problem: Is there a constant $C > 0$ such that every set $s \subset E_2$ of measure $> C$ contains the vertices of a triangle area? Problem: Given a countable subset A of $[0, 1]$; estimate the largest possible measure of a subset of $[0, 1]$ which does not contain a set similar to A.

Problem: A set S in an Euclidean space of finite dimension is said to be Ramsey if for every $k \in N$ there is an n_k such that if E_{n_k} is decomposed into k disjoint sets A_i then S is contained in one of these sets $A_1, \ldots, A_k$; is every obtuse angled triangle Ramsey? Is regular pentagon Ramsey? Problem: Is there a set S of power c in Hilbert space such that every equinumerous subset S_1 contains an equilateral triangle (resp. an infinite dimensional regular simplex)? Many other questions are discoussed in the present paper announcing that he matter will be extensively discussed along with other questions in a forthcoming book written jointly by the author and George Purdy.

:Kurepa

Classification:
04A05 Relations, functions
04A10 Ordinal and cardinal numbers; generalizations
04A20 Combinatorial set theory
05A17 Partitions of integres (combinatorics)
28A99 Classical measure theory
00A07 Problem books

Keywords:
combinatorial problems in set theory; different distances; subsets of Euclidean space; partitions; Hilbert space; measures