Zbl 417.10047
Erdős, Paul; Hardy, G.E.; Subbarao, M.V.
On the Schnirelmann density of k-free integers. (In English)
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For each integer $k \geq 1$ let $Q_x(x)$ be the number of k-free integers not exceeding x, and let the asymptotic and Schnirelmann densities of the set of k-free integers be, respectively, $d_k = \lim_{x\to\infty} Q_x(x)/x$ and $d_k = \inf_{n\geq 0} Q_x(n)/n$. *H.M.Stark* has shown [Proc. Am. Math. Soc. 17, 1211-1214 (1966; Zbl 144.28205)] that $d_k < D_k$ for all $k > 1$, hence that $d_k = Q_k(n_k)/n_k$ for at least one integer n_k. *P.H.Diananda* and *M.V.Subbarao* have proved [Proc. Amer. Math. Soc. 62, 7-10 (1977; Zbl 346.10026)] $d_k > 1 - 2^{-k} - 3^{-k} - 5^{-k}$ and several related results. It is now proved that

$$\{d_k - (1 - 2^{-k} - 3^{-k} - 5^{-k})\}//D_k - d_k) = o(2/3)^k \to 0 \text{ as } k \to \infty,$$

hence that d_k is always closer to $1 - 2^{-k} - 3^{-k} - 5^{-k}$ than to D_k. A table of values of n_k and $Q_x(n_k)$, $1 \leq k \leq 75$ is included in the paper, and several conjectures and problems are put forth.

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11B83 Special sequences of integers and polynomials
11N05 Distribution of primes

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