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Erdős, Paul; Katai, I.
On the growth of some additive functions on small intervals. (In English)
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Let $g : \mathbb{N} \to \mathbb{R}$ denote a non-negative strongly additive function, and let $f_k(n) = \max\{g(n+j) : j = 1, \dots, k\}$. The authors give conditions which imply that for every $\varepsilon > 0$ and every k_0 the inequality $f_k(n) < (1+\varepsilon)f_k(0)$ holds for $k \geq k_0$ and all but $\delta(\varepsilon, k_0)x$ integers n in $[1, x]$, $\delta(\varepsilon, k_0) \to 0$ for $k_0 \to \infty$. Some questions concerning the necessity of the conditions remain open. The main part of the paper is devoted to the special case $g = \omega$, where $\omega(n)$ denotes the number of distinct prime factors of $n \in \mathbb{N}$. Let

$$0_k(n) = \max\{\omega(n+j) : j = 1, \dots, k\}, o_k(n) = \min\{\omega(n+j) : j = 1, \dots, k\}.$$

The authors prove, by use of Brun's sieve, that for every $\varepsilon > 0$ the inequalities ($\log_2 = \log\log$)

$$0_k(n) \geq (1-\varepsilon)\rho\left(\frac{\log k}{\log_2 n}\right)\log_2 n, o_k(n) \leq (\overline{\rho}\left(\frac{\log k}{\log_2 n}\right) + \varepsilon)\log_2 n$$

hold for every $k \geq 1$ apart from a set of n's having zero density. Here ρ, $\overline{\rho}$ are defined as the inverse functions of Ψ with $\Psi(r) = r\log\frac{r}{e} + 1$ for $r \geq 1$ resp. $0 < r \leq 1$, $\overline{\rho}(u) = 0$ for $u \geq 1$. This result corresponds to similar upper resp. lower bounds obtained by *I.Kátai* [Publ. Math., Debrecen 18, 171-175 (1971; Zbl 261.10029)].

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11N05 Distribution of primes
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