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Erdős, Paul; Saffari, B.; Vaughan, R.C.

On the asymptotic density of sets of integers. II. (In English)

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[Part I, cf. ibid. 13, 475-485 (1976; Zbl 333.10039)]

Let A and B be a pair of direct factors of N^*, the set of positive integers; that is a pair of subsets A and B of N^* such that every $n \in N^*$ can be written uniquely as $n = a \cdot b$, with $a \in A$ and $b \in B$. Let $S \subset N^*$ and $d(S)$ denote the asymptotic density of S whenever it exists.Let $H(S) = \sum_{n \in s} \frac{1}{n}$. It has been shown by Saffari that in the convergent case, the sets A and B habe asymptotic densities: $d(A) = \frac{1}{H(B)}$ and $d(B) = \frac{1}{H(A)}$. In this paper the authors settle (in the affirmative) the first two open problems stated by Saffari. In fact they prove: Theorem 1. The direct factors A and B have asymptotic densities in the divergent case $H(A) = H(B) = \infty$ and $d(A) = 0$. Theorem 2. In the divergent case $H(A) = H(B) = \infty$, we have $\sum_{b \in A} \frac{1}{b} = \sum_{b \in B} \frac{1}{p} = \infty$.

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