The Cartesian product of two hamiltinian graphs is always hamiltonian. For directed graphs, the analogous statement is false. We show that the cartesian product $C_{n_1} \times C_{n_2}$ of directed cycles is hamiltonian if and only if the greatest common divisor (g.c.d.) d of n_1 and n_2 is at least two and there exist positive integers d_1, d_2 so that $d_1 + d_2 = d$ and g.c.d. $(n_1, d_1) = g.c.d. (n_2, d_2) = 1$. We also discuss some number- theoretic problems motivated by this result.

Classification:

05C45 Eulerian and Hamiltonian graphs

05C99 Graph theory

11P81 Elementary theory of partitions

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Cartesian product; Hamiltonian graphs; directed graphs; directed cycles