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On differences and sums of integers. I. (In English)

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A set  $B = \{b_1, b_2, \dots, b_i\} \subset \{1, 2, \dots, N\}$  is a difference intersector set if for any set  $A = \{a_1, a_2, \dots, a_j\} \subset \{1, 2, \dots, N\}, j = \varepsilon N$  the equation  $a_x - a_y = b$ has a solution. The notion of a sum intersector set is defined similary. Using exponential sum techniques, the authors prove two theorems which in essence imply that a set which is well-distributed within and amongst all residue classes of small modules is both a difference and a sum intersector set. The regularity of the distribution of the non-zero quadratic residues (mod p) allows the theorems to be used to investigate the solubility of the equations  $\left(\frac{a_x - a_y}{p}\right)$ +1,  $\left(\frac{a_r-a_s}{p}\right)=-1$ ,  $\left(\frac{a_t-a_u}{p}\right)=+1$ , and  $\left(\frac{a_v-a_w}{p}\right)=-1$ . The theorems are also used to establish that "almost all" sequences form both difference and sum intersector sets.

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## Classification:

11B83 Special sequences of integers and polynomials

11B13 Additive bases

11P99 Additive number theory

11D85 Representation problems of integers

11L03 Trigonometric and exponential sums, general

## Keywords:

difference intersector set; sum intersector set; distribution quadratic residues; sequence of integers