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Erdős, Paul
Some unconventional problems in number theory. (In English)
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This fascinating miscellany covers a great many topics and the best this reviewer can do is attempt to give a taste of the paper. As stated by the author the paper mostly deals with arithmetic functions, primes, divisors, sieve processes and consecutive integers. Let f be an arithmetic function. The integer n is called a barrier for f if $m+f(m)\leq n$ for every $m<n$. For $n=\prod p_i^{\alpha_i}$ put $d_0(n)=\prod \alpha_i$. Then $d_0(n)$ has infinitely many barriers, that is there are infinitaly many n such that $m+d_0(m)\leq n$ for every $m<n$. In fact the density of integers n which are barriers for d_0 is positive. An outline of the proof of this Theorem is given including the following observation. Let $\varepsilon>0$, k be a sufficiently large integer and A be a multiple of $p_1,p_2,\dots,p_k$. Then the density of integers t for which $d_0(tA^k-i)>i$, for some i with $l\leq i\leq k$, is less than $\frac{1}{2}\varepsilon$. Among others the following problem is discussed. Let $p(m)$ denote the least prime factor of m and put $F(n)=\max\{m+p(m): l\leq m<n,\ m \text{ composite}\}$. Is it true that $F(n)\leq n$ for infinitely many n?

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Classification:
11N37 Asymptotic results on arithmetic functions
11-02 Research monographs (number theory)
11A25 Arithmetic functions, etc.
00A07 Problem books

Keywords:
arithmetic function; consecutive integers; problems; primes; divisors sieve processes