Zbl 383.30001

Erdős, Paul; Hwang, J.S.

On a geometric property of Lemniscates. (In English)

Aequationes Math. 17, 344-347 (1978). [0001-9054]

Motivated by a property of polynomials of a complex variable, the authors prove the theorem below and discuses related open questions. Theorem. Let $p_n(w, w_k) = \prod_{k=1}^n |w - w_k| \quad (w, w_k \in \mathbb{R}^3) \text{ and } E(p_n) = \{w : p_n(w, w_k) \le 1\}.$ If $p_n(w, w_k)$ and $p_n(w, w_k^*)$ are such that $E(p_n) \subseteq E(p_n^*)$ and if all the zeros w_k of p_n lie on the same plane, then $p_n(w, w_k) \equiv p_n^*(w, w_k^*)$. Moreover, the hypothesis $E(p_n) \subseteq E(p_n^*)$ is not sufficient to deduce $p_n = p_n^*$. [For further properties of products $p_n(w, w_k)$, see J.B.Diaz and D.B.Schaffer, Appl. Anal. 6, 109-117 (1977; Zbl 346.30003).]

A. Giroux

Classification:

30C10 Polynomials (one complex variable)