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Let $f(z) = \sum_{k=0}^{\infty} a_k z^k, a_0 > 0, a_k \geq 0 (k \geq 1)$ be an entire function such that $l < \limsup_{r\to\infty} \frac{\log\log M(r)}{\log\log r} = \rho + 1 < \infty$ and $\limsup_{r\to\infty}(\inf) (\log r)^{-\rho-1} \log M(r) = \alpha(\beta)$ where $M(r) = \max_{\|z\|=r} \|f(z)\|$. The authors in Adv. Math. 21, 78-109 (1976; Zbl 334.30019) showed that if (*) $5 < 2\beta < 2\alpha < \infty$ then for every sequence of polynomials $\{P_n(x)\}_{n=0}^{\infty}$ of degree at most n, $\liminf_{n\to\infty} \delta_n^{n^{-1-\rho^{-1}}} \geq e^{-1}$ where $\delta_n = \left\| \frac{1}{f(x)} - \frac{1}{P_n(x)} \right\|_{L_\infty[0,\infty]}$.

In the present paper, they show that if (*) is replaced by (**) $0 < \beta \leq \alpha < \infty$, then for every polynomial $P_n(x)$ and $Q_n(x)$ of degree at most n, $\liminf_{n\to\infty} \triangle_n^{n^{-1-\rho^{-1}}} \geq G$ where $\triangle_n = \left\| \frac{1}{f(x)} - \frac{P_n(x)}{Q_n(x)} \right\|_{L_-\infty[0,\infty]}$ and $G = \exp\left\{ -\left(\frac{2}{\beta}\right)^{1/\rho} \left[\alpha - 1 + \left(\frac{2\alpha}{\beta}\right)^{1/(\rho+1)} \right] \right\}$.

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30E10 Approximation in the complex domain