Zbl 374.05047
Erdős, Paul; Meir, A.
On total matching numbers and total covering numbers of complementary graphs. (In English)
Discrete Math. 19, 229-233 (1977). [0012-365X]

A vertex u of a graph G is said to cover itself, all incident edges, and all adjacent vertices. An edge uv of G covers itself, u and v, and all adjacent edges. A subset S of $V(G) \cup E(G)$ is called a total cover if the elements of S cover G. Two elements of $V(G) \cup E(G)$ are said to be independent if neither covers the other. Define $\alpha_2(G) = \min |S|$, where the min is taken over all total covers S of G, and $\beta_2(G) = \max |T|$, where the max is taken over all subsets T of $V(G) \cup E(G)$ whose elements are pairwise independent. The following theorems are presented.
Theorem 1: If G is a graph on n vertices, then

$$2\{n/2\} \leq \beta_2(G) + \beta_2(\overline{G}) \leq \{3n/2\}.$$

The upper bound is best possible for all n, the lower bound is best possible for all $n \neq 2(\mod 4)$.
Theorem 2: If G is a graph on n vertices, then

$$\{n/2\} + 1 \leq \alpha_2(G) + \alpha_2(\overline{G}) \leq \{3n/2\}.$$

The upper bound is best possible for all n, the lower bound is best possible for odd n.
Theorem 3: If G is a connected graph on $n \geq 2$ vertices, then

$$\alpha_2(G) + \beta_2(G) \leq n + \{n/2\}/2.$$

This bound is best possible.

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Classification:
05C99 Graph theory
05B40 Packing and covering (combinatorics)