Zbl 373.60035

Erdős, Pál; Revesz, Pál

On a problem of T. Varga. (In Hungarian)

Mat. Lapok 24(1973), 273-282 (1975). [0025-519X]

Toss a fair coin n times. What is the length Z_n of the longest head-run? This question was initiated by an interesting teaching experiment of T. Varga. The problem can be formulated the following way. Let $X_1, X_2, \dots$ be i.i.d. rv's with $p(X_1 = 0) = p(X_2 = 1) = 1/2$, $S_0 = 0$, $S_k = X_1 + \dots + X_k$, $I(n,k) = \max_{0 \le i \le n-k}(S_{i+k} - S_i)$. Then Z_n is the largest integer for which $I(n, Z_n) = Z_n$. *P.Erdős* and *A.Rényi* [J. Anal. Math. 23, 103-111 (1970; Zbl 225.60015] proved that if $0 < c_1 < 1 < c_2 < \infty$, then for almost all $\omega \in \Omega$ (the basic space) there exists a finite $n_0 = n_0(\omega, c_1, c_2)$ such that for $n \ge n_0$, $[c_1 \log n] \le Z_n < [c_2 \log n]$ (log is with base 2, [.] means integer part x the smallest integer $\ge x$). Let $\alpha_n(3) = [\log n - \log\log\log n] + 3$, $\alpha_n(0) = \{\log n - \log\log\log n\}$. Here the authors prove that for almost all ω there exists n_0, such that $\alpha_n(3) \le Z_n$, for $n \ge n_0$. On the other hand, for almost all ω there exists an infinite sequence $n_k(\omega)$ such that $Z_{n_k} < \alpha_{n_k}(0)$. Clearly Z_n can be much larger than $\alpha_n(3)$ for some n. Concerning the largest possible values of Z_n, the following results are proved. If $\{\gamma_n\}$ is a sequence of positive numbers such that $\Sigma 2^{-\gamma_n} = \infty$, then for almost all ω there exists an infinite sequence $n_k = n_k(\omega, \{\gamma_n\})$ of integers such that $Z_{n_k} \ge \gamma_{n_k}$. But if $\Sigma 2^{-\gamma_n} < \infty$, then for almost all ω there exists $n_0 = n_0(\omega, \{\gamma_n\})$ such that $Z_n < \gamma_n$ for $n \ge n_0$. All the mentioned results are then generalised for the length of the longest run containing a fixed number of tails. The result in this paper are formulated in nonprobabilistic language, concerning 1-runs in the dyadic expansion of a number $0 \le x \le 1$. A list of unsolved problems is also included. Note that in the English version of the paper [Top. Inf. Theory, Keszthely 1975, Colloq. Math. Soc. János Bolyai 16, 219-228 (1977; Zbl 362.60044)] the first two results appear in a finer form. Namely $[\log n - \log\log\log n + \log\log e - 2 - \varepsilon]$ and $[\log n - \log\log\log n + \log\log e - 1 + \varepsilon]$ play respectively the roles of $\alpha_n(3)$ and $\alpha_n(0)$ where $0 < \varepsilon$ is arbitrarily small.

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Classification:
60F15 Strong limit theorems