Zbl 372.41008

Erdős, Paul; Newman, D.J.; Reddy, A.R.

Approximation by rational functions. (In English)

J. London Math. Soc., II. Ser. 15, 319-328 (1977).

This paper contains eight theorems on the rational approximation of e^{-x} . We cite one of them by way of an example: "Let $p(x)$ and $q(x)$ be any polynomials of degress at most $n-1$ where $n \geq 2$. Then we have

$$\left\| e^{-x} - \frac{p(x)}{q(x)} \right\|_{l_\infty(N)} \geq \frac{(e-1)^n e^{-4n} 2^{-7n}}{n(3+2\sqrt{2})^{n-1}}.'',$$

(N is the set of non-negative integers). Another theorems is a result of the same type for $\left\| e^{-x} - \frac{p(x)}{q(x)} \right\|_{L_\infty[0,1]}$, with the restriction on p(x) that its coefficients are non-negative. It should have been mentioned that the rational function $r_{m,n}(x)$ with denominator of degree m and numerator of degree n (not m), both defined by an integral, for which it is shown that, theorem 2,

$$\left\| e^{-x} - r_{m,n}(x) \right\|_{L_\infty[0,1]} \leq \frac{m^n n^n}{(m,n)^{m+n}(m+n)!},$$

is in fact the Padé approximant of e^{-x}. From the various results applied during the proofs of the eight theorems we mention Lagrange's interpolation theorem, interpolation polynomials from the calculus of differences and a lemma of the second author which says that $[p(x)]^{\frac{1}{n}}$ is concave on $[a,b]$ when the polynomial p has degree at most n, has only real zeros and $p(x) < 0$ on $[a,b]$.

H.Jager

Classification:
41A20 Approximation by rational functions