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Erdős, Paul; Newman, D.J.
Bases for sets of integers. (In English)
J. Number Theory 9, 420-425 (1977). [0022-314X]

If every member of a set A of positive integers can be expressed in the form $b+b'$ for some b, $b' \in B$, then the set B of non-negative integers is called a basis for A. The problem is to find bounds on the size m of the smallest basis B for a set A of type (n,N), i.e. for a set A with n elements, the largest of which is N. It is easy to show that $n^{\frac{1}{2}} \le m \le \min\{n+1,(4N+1)^{\frac{1}{2}}\}$. The authors show that, for most sets A of type (n,N),

$$m > \min\left\{\frac{n}{\log N}, \frac{\sqrt{N}}{2}\right\},$$

so that, for most sets A, the actual value of m is closer to the upper bound than to the lower bound. A special study is then made of the sequence of squares, $A=\{1^2,2^2,\dots,n^2\}$, and it is shown that this sequence is untypical since, for arbitrarily large M, $n^{2/3-\varepsilon} \le m \le n/\log^m n$. (There is an unfortunate misprint at this point.) Closing remarks include the observation that the value of m depends critically not just on n and N, but on the arithmetical character of the sequence.

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Classification:
11B13 Additive bases
11B83 Special sequences of integers and polynomials