Zbl 353.05045
Bollobás, Béla; Erdős, Paul
An extremal problem of graphs with diameter 2. (In English)
Math. Mag. 48, 281-283 (1975). [0025-570X]

For integers p and k such that $1 \leq k < p$, $m(p, k)$ is the minimum number of edges for which there exists a graph on p vertices, every pair of which are connected by at least k paths of length 1 or 2. *U.S.R.Murty* [On critical graphs of diameter 2, Math. Mag. 41, 138-140 (1968; Zbl 167.22102)] proved that if $p \geq (1/2)(3 + \sqrt{5})k$, then $m(p, k) = \binom{p}{2} - \binom{p-k}{2}$ and characterized the unique extremal graph. The Theorem of this paper states that if $1 < c < (1/2)(3+\sqrt{5})$, then $m([ck], k) = c^{3/2}k^2/2 + o(k^2)$. Several proofs are provided.

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Classification:
05C35 Extremal problems (graph theory)