Zbl 337.10001

Erdős, Paul

Problems and results on number theoretic properties of consecutive integers and related questions. (In English)

Proc. 5th Manitoba Conf. numer. Math., Winnipeg 1975, 25-44 (1976).

[For the entire collection see Zbl 327.00009.]

This is an entertaining survey paper on the factorization of factorials and binomial coefficients and related questions. Outlines of proofs of three new results are given. The first result states that to every $\epsilon>0$ and $\eta>0$ there is a $k=k(\epsilon,\eta)$ so that the upper density of integers n for which the greatest prime factor of $\prod_{i=1}^k (n+1)$ is less than $n^{\frac{1}{2}-\epsilon}$ is at most η . The third result says that for $n>n_0(\epsilon)$ the equation (*) $n!=\prod_{i=1}^t (m+1)$ has no solutions for $m<(2-\epsilon)^n$. It is conjectured that in the first result $n^{\frac{1}{2}-\epsilon}$ can be replaced by $n^{1-\epsilon}$ and that equation (*) has only trivial solutions except for 6!=8.9.10. The constant mentioned in relation to Catalan's conjecture on consecutive powers and attributed to the reviewer should be read as an indication what kind of bound can be obtained by using the Gel'fond-Baker method. In the paper [Acta arithmetica 29, 197-209 (1976; Zbl 286.10013)] it is only proved that there exists an effectively computable constant.

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Classification:

11-02 Research monographs (number theory)

11A41 Elemementary prime number theory

11D57 Multiplicative and norm form diophantine equations

00A07 Problem books