
Zbl 329.10005
Erdős, Paul; Simonovits, Miklos
On a problem of Hirschhorn. (In English)
Am. Math. Mon. 83, 23-26 (1976). [0002-9890]

In 1973 *M. D. Hirschhorn* gave the following problem [Amer. math. Monthly 80, 675-677 (1973; Zbl 266.10009)]: Let $q_1 > 1$ be given and $q_{n+1} - q_n = \prod_{i\le n}\left(1-\frac{1}{q_i}\right)^{-1}$. Then does it necessarily follow that $q_n = (1+o(1))n\log n$? In this note, the authors settle the problem in the affirmative. The background of this problem is that, if p_n denotes the n-th prime, then the well-known sieve method gives that the number of integers between a and b which are not divisible by any of $p_1,\dots,p_n$ is approximately $(b-a)\prod_{i\le n}\left(1-\frac{1}{p_i}\right)$. The interval $(p_n, p_{n+1}]$, contains exactly one prime, i.e., exactly one integer not divisible by any $p_i (i\le n)$. This suggests that $p_{n+1} - p_n = \prod_{i\le n}\left(1-\frac{1}{p_i}\right)^{-1}$. We know in this special case by the prime number theorem that $q_n = (1+o(1))n\log n$.

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Classification:
11A41 Elemementary prime number theory
11B37 Recurrences