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Bemerkungen zu einer Aufgabe in den Elementen.

Remarks on a problem in the elements. (In German)

Arch. der Math. 27, 159-163 (1976).

Let p denote an odd prime and $\ell(p)$ the order of 2 mod p. Write $E(r)$ for the number of odd primes p with $\ell(p)=r$ and $A(x,\delta)$ for the number of odd primes p with $p\le x$ and $\ell(p)>p^\delta$, $0<\delta<1$. *Jaeschke* and *Bundschuh* (Aufgabe 618 in "Elemente der Mathematik", 1971, hence the uniformative title of the present paper) proved that

$$\text{(i)}\quad E(r)\le\frac{r\log 2}{\log r},\quad \text{(ii)}\quad A(x,\delta)=(1+o(1))\frac{x}{\log x},\quad 0<\delta<\frac{1}{2},$$

$$\text{(iii)}\quad A(x,\frac{1}{2})\ge(1-\log 2+o(1))\frac{x}{\log x}.$$

In a few lines, using the Chinese remainder theorem and a weak form of Stirling's formula, (i) is sharpened to

$$E(r)\le(\frac{1}{2}+o(1))\frac{r\log 2}{\log r}.$$

The main result of the paper is that (ii) also holds for $\delta=\frac{1}{2}$. The proof of this is rather complicated and uses among other things an estimate from the sieve method of Brun. There are some conjectures on $E(r)$ and $A(x,\delta)$ which seem very difficult to prove. To mention one of them: $E(r)=o(r^\epsilon)$, $\epsilon>0$.

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Classification:
11A05 Multiplicative structure of the integers
11N37 Asymptotic results on arithmetic functions