Zbl 318.05110

Bollobás, Béla; Erdős, Paul; Simonovits, M.

On the structure of edge graphs. II. (In English)

J. London Math. Soc., II. Ser. 12, 219-224 (1976).

Denote by G(n, m) a graph with n vertices and m edges and by $K_d(t)$ the complete d-partite graph with t vertices in all classes. The following theorem is proved: (a) There is an absolute constant $\alpha > 0$ such that if 0 < c < 1/d and $m > (1 - 1/d + c)n^2/2$, then every G(n, m) contains a $K_{d+1}(t)$, where

(1)
$$t = \left[\frac{\alpha \log n}{d \log(1/c)}\right].$$

(b) Given an integer $d \ge 1$ there exists a constant $\epsilon_d > 0$ such that if $0 < c < \epsilon_d$ and $n \ge n(d,c)$ is an integer then there exists a graph G(n,m) satisfying (1) which does not contain a $K_{d+1}(t)$ with

$$t = \left[5 \frac{\log n}{\log(1/c)} \right].$$

This is a sequel to the article of *B. Bollobás* and *P. Erdős* in Bull. London math. Soc. 5, 317- 321 (1973; Zbl 277.05135).

 $\check{S}t.Zn\acute{a}m$

Classification:

05C35 Extremal problems (graph theory)