Zbl 312.10003

Erdős, Paul

On abundant-like numbers. (In English)

Can. Math. Bull. 17, 599-602 (1974).

Let $2 - p_1 < p_2 < \dots$ be the sequence of primes. Denote by $n_k^{(c)}$ the smallest integer for which p_k is the smallest prime divisor of $n_k^{(c)}$ and $\sigma(n_k^{(c)} \ge c n_k^{(c)})$ where $\sigma(n)$ denotes the sum of the divisors of n. From the reviewer's solution of a problem proposed by the author [Canadian math. Bull. 16, 144 (1973)] it follows that there are only a finite number of squarefree integers which are $n_k^{(c)}$'s for some $c \geq 2$ (maybe only the integer 6). The author now proves that $n_k^{(2)}$ is cubefree for all sufficiently large k. The proof depends on a method developed by Ramanujan. The situation for 1 < c < 2 is much more complicated. It is shown e.g. that the sets of numbers c for which $n_k^{(c)}$ is infinitely often squarefree resp. not squarefree are both dense in (1,2).

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Classification:

11A05 Multiplicative structure of the integers

11A25 Arithmetic functions, etc.