Zbl 306.05121
Bollobás, Béla; Erdős, Paul; Szemeredi, E.
On complete subgraphs of r-chromatic graphs. (In English)
Discrete Math. 13, 97-107 (1975). [0012-365X]

Let $G_r(n)$ be an r-chromatic graph with n vertices in each colour class. Suppose $G = G_3(n)$, and $\delta(G)$, the minimal degree in G, is at least $n + t(t \geq 1)$. We prove that G contains at least t^3 triangles but does not have to contain more than $4t^3$ of them. Furthermore, we give lower bounds for s such that G contains a complete 3-partite grpah with s vertices in each class. Let

$$f_r(n) = \max\{\delta(G) : G = G_r(n), \quad G \text{ does not contain a complete graph with } r \text{ vertices}\}.$$

We obtain various results on $f_r(n)$. In particular, we prove that if $c_r = \lim_{n \to \infty} f_r(n)/n$, then $\lim_{r \to \infty}(c_r - (r - 2)) \geq 1/2$ and we conjecture that equality holds. We prove several other results and state a number of unsolved problems.

Classification:
05C35 Extremal problems (graph theory)
05C15 Chromatic theory of graphs and maps