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On the prime factors of $\binom{2n}{n}$. (In English)

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The present paper is devoted to a quantitative study of the factors of the binomial coefficient $B_n = \binom{2n}{n}$. Among the results obtained are the following: (1) for any two odd primes p and q, $(B_n, pq) = 1$ for infinitely many integers n; (2) if $f(n) = \sum 1/p$ where the summation is over all primes p such that $p \leq n$ and $p \nmid B_n$, then $\lim_{x\to\infty} x^{-1} \sum_{n=1}^{x} f(n) = \sum_{k=2}^{\infty} \log k/2^k$; (3) if p is a fixed prime and

$$S = \{n \leq x : p^{\alpha} | B_n \text{ and } p^{\alpha} \notin (n^{1/2-\epsilon}, n^{1/2+\epsilon})\},$$

then the cardinality of S is $0(x)$.

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Classification:
11B39 Special numbers, etc.
11A41 Elememtary prime number theory
05A10 Combinatorial functions