Zbl 274.10043
Erdős, Paul; Hall, R.R.
On the distribution of values of certain divisor functions. (In English)
J. Number Theory 6, 52-63 (1974). [0022-314X]

Let $\{\epsilon_d\}$ be a sequence of non-negative numbers and $f(n) = \sum\{\epsilon_d : d \mid n\}$. The authors investigate under what circumstances there exists a continuous distribution function $F(c)$ such that $F(c) \to 0$ as $c \to \infty$ and for each fixed c, $\text{card}\{n < x : f(n) > c\} \sim F(c)$. They show that it is sufficient that $\sum\{1/p : \epsilon_p > 0\} = \infty$ and for some fixed $\beta > 0$,

$$(1) \qquad 0 \leq \epsilon_d \leq 2^{-\log\log d - (1+\beta)(2\log\log d.\,\log\log\log\log d)^{1/2}}.$$

The authors also obtain the result $F(c - \delta) - F(c) \ll (\log 1/\delta)^{-1/2}$ uniformly for all c and $\delta < 1/2$ in the special cases $\epsilon_d = (\log d)^{-\alpha}$, $(\alpha > \log 2, d \geq 2)$ or when (1) holds with equality on the right. The conditions $\alpha > \log 2$, $\beta > 0$ are best possible in their contexts.

Classification:

11N37 Asymptotic results on arithmetic functions