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Brown, William G.; Erdős, Paul; Simonovits, Miklos

Extremal graph problems for directed graphs. (In English)

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We consider directed graphs without loops and multiple edges, where the exclusion of multiple edges means that two vertices cannot be joined by two edges of the same orientation. Let $L_1, \dots, L_q$ be given digraphs. What is the maximum number of edges a digraph can have if it does not contain any L_i as a subgraph and has given number of vertices? We shall prove the existence of a sequence of asymptotical extremal graphs having fairly simple structure. More exactly: There exists a matrix $A = (a_{i,j})_{i,j\le r}$ and a sequence $\{S^n\}$ of graphs such that (i) the vertices of S^n can be divided into classes $C_1, \dots, C_r$ so that, if $i \neq j$, each vertex of C_i is joined to each vertex of C_j by an edge oriented from C_i to C_j if and only if $a_{i,j} = 2$; the vertices of C_i are independent if $a_{i,i} = 0$; and otherwise $a_{i,i} = 1$ and the digraph determined by C_i is a complete acyclic digraph; (ii) S^n contains no L_i but any graph having $[\epsilon n^2]$ more edges than S^n must contain at least one L_i. (Here the word graph is an “abbreviation” for “directed graph or digraph”.)

Classification:
05C20 Directed graphs (digraphs)
05C35 Extremal problems (graph theory)