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On a new law of large numbers. (In English)

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We shall prove first (in §2) the new law of large numbers for the simplest special case, that is for independent repetitions of a fair game. For this special case the theorem can be stated as follows: if the game is played N times, the maximal average gain of a player over $[C\log_2 N]$ consecutive games ($C\geq 1, [x]$ denotes the integral part of x), tends with probability one to the limit α, where α is the only solution in the interval $0<\alpha\leq 1$ of the equation

$$\frac{1}{C}=1-\left(\frac{1+\alpha}{2}\right)\log_2\left(\frac{2}{1+\alpha}\right)-\left(\frac{1-\alpha}{2}\right)\log_2\left(\frac{2}{1-\alpha}\right).$$

In §3 we generalize this result to an arbitrary sequence η_n ($n=1,2,\dots$) of independent, identically distributed random variables with expectation 0, the common distribution of which satisfies the condition, that its moment-generating function $\varphi(t)=E(e^{\eta_n t})$ exists in an open interval around the origin. We prove that for every α in a certain interval $0<\alpha<\alpha_0$ one has

$$(*)\qquad P\left(\lim_{N\to+\infty}\max_{0\leq n\leq N-[C\log N]}\frac{\eta_{n+1}+\eta_{n+2}+\dots+\eta_{n+[C\log N]}}{[C\log N]}=\alpha\right)=1,$$

where $C=C(\alpha)$ is defined by the equation $e^{-(1/C)}=\min_t\varphi(t)e^{-\alpha t}$. In §4 we discuss the special case of Gaussian random variables, in which case our result is essentially equivalent to a previous result of *P. Lévy* about the Brownian movement process. In §5 we give as an application of the result of §3, a new proof of the theorem of *P. Bártfai* on the "stochastic geyser problem", using the fact that the functional dependence between C and α in (*) determines the distribution of the variables uniquely (Theorem 3). The result of §2 can also be applied in probabilistic number theory; as a matter of fact it was such an application which led the first named author to raise the problem which is solved in the present paper.

Classification:
60F15 Strong limit theorems
60J65 Brownian motion