Zbl 205.34902

Articles of (and about)

Erdős, Paul

On the distribution of the convergents of almost all real numbers. (In English) J. Number Theory 2, 425-441 (1970). [0022-314X]

Let $n_1 < n_2 < \dots$ be an infinite sequence of integers. The necessary and sufficient condition that for almost all α infinitely many n_i should occur among the convergents of α is that $\sum_{i=1}^{\infty} \varphi(n_i)/n_i^2 = \infty$, where φ is the Euler φ -function. The necessity is obvious, but the proof of the sufficiency is complicated. In fact it is proved that if $\sum_{i=1}^{\infty} \varphi(n_i)/n_i^2 = \infty$ then for every $\epsilon > 0$ and almost all α

$$|\alpha - a/n_i| < \epsilon/n_i^2, \quad (a, n_i) = 1$$

has infinitely many solutions.

Classification:

11J25 Diophantine inequalities