Zbl 204.00905
Erdős, Paul; Rado, R.
Partition relations and transitivity domains of binary relations (In English)
J. Lond. Math. Soc. 42, 624-633 (1967).

The main theorem is Theorem 2: For all positive integers m and n, for some positive integer $l(m,n)$, for each ordinal number α, $\omega_\alpha l(m,n) \to (m, \omega_\alpha n)^2$; if $l_\alpha(m,n)$ is the least such $l(m,n)$ for a given α, then $\gamma \mapsto (m, \omega_\alpha n)^2$ for each $\gamma > \omega_\alpha l_\alpha(m,n)$, and

$$l_\alpha(m,n) \leq (2n-3)^{-1}[2^{m-1}(n-1)^m + n - 2];$$

if $m > 1$, then $l_0(m,n) \leq l_\alpha(m,n)$.
This generalizes some of Theorem 1, which handles the case $\alpha = 0$ and was proved by the same authors [Bull. Am. Math. Soc. 62, 427-489 (1956; Zbl 071.05105), Theorem 25]; Theorem 1 includes also a characterization of $l_0(m,n)$. Among other results, Theorem 4 is an extension (the statement of which is not the obvious one) to infinite a of a result attributed to R. Stearns: If a is a finite cardinal, if $\prec$ is a relation trichotomous on a set S, then $\prec$ is transitive on some subset of S having cardinal a provided that $|S| \geq 2^{n-1}$.
{Reviewer's remarks:
(1) It is easily seen that $l_0(m,2)$ as characterized by Theorem 1 is the least integer l such that for each set S with cardinal $\geq l$, for each relation $\prec$ trichotomous on S, $\prec$ is transitive on some subset of S having cardinal a. Specializing the estimate for $l_\alpha(m,n)$ in Theorem 2 to $n = 2$ yields the Stearns result.
(2) Theorem 1 is slightly misstated. If $m = 1$, "$\gamma \mapsto (m, \omega_0 n)^2$" should be changed by replacing γ by $\omega_0(l_0(m,n) - 1)$.
(3) In the footnote on p. 625 "$(n-1)^\mu$" should be replaced by "$(2(n-1))^\mu$".
(4) On p. 627, (i) is not quite adequate but becomes so on replacing "for $x \in A'_\beta$", by "for each $x \in A'_\beta$ and $|U_0(x)A_\gamma| = \aleph_\alpha$ for some $x \in A_\beta$", and (ii) is not quite adequate but becomes so on inserting "and $|U_0(x)A_\gamma| < \aleph_\alpha$ for some $x \in A_\beta$" after "$|U_0(\bar{x})A_\gamma| = \aleph_\alpha$". [The iteration of the operators O_λ then becomes adequate for the task at hand.] The last sentence of the third paragraph of (ii) appears to be an inaccurate oversimplication. [It is clear from the rather involved proof of Theorem 2, to which these points attach, that these inaccuracies were not in the original thinking things through. There are only a few others, which are more easily spotted.]
(5) In line with (1) above and the discussion of $l_0(m,n)$ on p. 624, the condition under (i) in Theorem 4 is best possible not only for $1 \leq a \leq 3$ but also for $1 \leq a \leq 4$}.

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Classification:
05D10 Ramsey theory
03E05 Combinatorial set theory (logic)
04A20 Combinatorial set theory