Zbl 159.06003

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On the solvability of some equations in dense sequences of integers (In English. RU original)

Sov. Math., Dokl. 8, 1160-1164 (1967); translation from Dokl. Akad. Nauk SSSR 176, 541-544 (1967). [0197-6788]

Let $a_1 < a_2 < ...$ be an infinite sequence of integers for which the equation $(a_i, a_j) = a_r$ is unsolvable for any set of distinct indices i, j, r. The authors prove that then

$$(1) \sum_{k=1}^{\infty} \frac{1}{a_k \log a_k} < \infty.$$

The proof is elementary but quite complicated and uses combinatorial arguments. In a previous paper [P. Erdős, J. London Math. Soc. 10, 126-128 (1935; Zbl 012.05202)] the following weaker result was proved: Assume that no a_i divides any other than (1) holds. In another paper [J. Math. Anal. Appl. 15, 60-64 (1966; Zbl 151.03502)] the authors point out that (1) does not hold if we assume that $[a_i, a_j] = a_r$ is unsolvable for any set of distinct indices i, j, r.

Classification:

11B83 Special sequences of integers and polynomials

11B75 Combinatorial number theory