Zbl 148.05402
Erdős, Pál
Remarks on a theorem of Zygmund (In English)
Proc. Lond. Math. Soc., III. Ser. 14 A, 81-85 (1965). [0024-6115]

We call a sequence of integers $n_1 < n_2 < \cdots$ a Zygmund sequence if whenever $|a_k| \to 0$, the power series

$$\sum_{k=1}^{\infty} a_k z^{n_k}$$

converges for at least one z with $|z| = 1$. It is known that any sequence $\{n_k\}$ satisfying $n_{k+1}/n_k > 1+c$ $(c > 0)$ is a Zygmund sequence, and that a Zygmund sequence con not contain arbitrarily long arithmetic progressions [cf. *J.-P. Kahane* (Zbl 121.30102)]. The author shows the following: Let $n_1 < n_2 < \cdots$ be a sequence which contains two subsequences $\{n_{k_i}\}$ and $\{n_{l_i}\}$, $1 \le i < \infty$, satisfying

$$k_i \to \infty, \quad k_i < l_i < k_{i+1}, \qquad l_i - k_i \to \infty, \quad (n_{l_i} - n_{k_i})^{1/(l_i-k_i)} \to 1.$$

Then the above sequences is not a Zygmund sequence.

M.Kinukawa

Classification:
30B10 Power series (one complex variable)

©European Mathematical Society & FIZ Karlruhe & Springer-Verlag