Zbl 146.27102

Erdős, Pál; Sarközy, A.; Szemeredi, E.

On the divisibility properties of sequences of integers. I (In English)

Acta Arith. 11, 411-418 (1966). [0065-1036]

Let $A = \{a_n\}$ be a sequence of integers; set $f(x) = \sum_{a_i|a_j, a_j \le x} 1$. The main result of this paper is Theorem 1. If A has positive upper logarithmic density c_1, then there exists c_2, depending on c_1 only, so that for infinitely many x, $f(x) > x \exp\{c_2(\log_2 x)^{1/2} \log_3 x\}$. On the other hand, there exists a sequence A of positive upper logarithmic density c_1, so that for all $x, f(x) < x \exp\{c_3(\log_2 x)^{1/2} \log_3 x\}$. (All c_i stand for positive constants and $\log_k x$ for the iterated logarithm.) The first inequality is proved using a purely combinatorial Theorem: Let $\mathcal{S}$ be a set of n elements and let $B_1, ..., B_2$, $z > c_4 2^n$ $(0 < c_4 < 1)$ be subsets of $\mathcal{S}$. Then, if $n > n_0(c_4)$, one of B's contains at least $\exp(c_5 n^{1/2} \log n)$ of the B's, where c_5 depends only on c_4. The second inequality is proved using a result of probabilistic number theory.

E.Grosswald

Classification:
11B83 Special sequences of integers and polynomials
11B05 Topology etc. of sets of numbers