Zbl 146.27101

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On a theorem of Behrend (In English)

J. Aust. Math. Soc. 7, 9-16 (1967).

A sequence $A = \{a_i\}$ of positive, increasing integers is called primitive if no term divides another. Let $\log_2 x = \log\log x$ and $f_A(x) = \sum_{a_i<x} a_i^{-1}$ and denote by c_i absolute, positive constants. *F.Behrend* (Zbl 012.05203) proved: If A is primitive, then there exists c_1 such that

$$f_A(x) < c_1 \log x(\log_2 x)^{1/2}.$$

Pillai showed that there exists c_2 such that for every x there is a finite primitive sequence A, with $a_i \leq x$ for which $f_A(x) > c_2 \log x(\log_2 x)^{-1/2}$, so that Behrend's theorem is, in this sense, best possible.

In the present paper it is shown that if A is infinite, Behrend's result can be improved to read

$$(^*) f_A(x) = o(\log x(\log_2 x)^{-1/2}).$$

However, no further improvement is possible, because, if $h(x) \to \infty$ arbitrarily slowly, then there exists a primitive sequence A, so that

$$\limsup_{x\to\infty} f_A(x)h(x)(\log_2 x)^{1/2}(\log x)^{-1} = \infty.$$

For a proof, the general case is first reduced to that of squarefree a_i's; then the author use a lemma, which, while technically elementary, has a rather long and difficult proof. A sharper form of $(^*)$ and some related topics are also discussed and proved.

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Classification:
11B83 Special sequences of integers and polynomials