Zbl 146.05304
Erdős, Pál
On the multiplicative representation of integers (In English)
Isr. J. Math. 2, 251-261 (1964). [0021-2172]

Let $b_1 < b_2 < \cdots$ be an infinite sequence of integers and $g(n)$ the number of solutions of $n = b_i b_j$. It is shown that if $g(n) > 0$ for all $n > n_0$ then $\limsup_{n\to\infty} g(n) = \infty$. A proof of the following result is outlined. Denote by $u_l(n)$ the smallest integer so that if $b_1 < \cdots < b_t \le n$, $t \le u_l(n)$ is any sequence of integers then forsome m, $g(m) \ge l$. If $2^{k-1} < l \le 2^k$ then

$$u_l(n) = (1 + o(1))n(\log\log n)^{k-1}/(k-1)!\log n.$$

{There are numerous minor misprints including the denominator in (6) which should be read as $N_r^{1/2^{r-1}}$.}

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Classification:
11A67 Representation systems for integers and rationals
11B75 Combinatorial number theory