Zbl 136.44901
Erdős, Pál; Moser, L.
On the representation of directed graphs as unions of orderings (In English)
Publ. Math. Inst. Hung. Acad. Sci., Ser. A 9, 125-132 (1964).

In this paper an $m \times n$ matrix R is considered in which each row consists of a permutation of the integers $1, 2, ..., n$. Such matrix is called the $m \times n$ R-matrix (or briefly the R-matrix). We define an oriented graph on the vertices $1, 2, ..., n$, in which there is an edge oriented from i to j provided i precedes j in a majority of the rows of R. If i precedes j as often as j precedes i, the vertices i, j are not joined by an edge. McGarvey [Econometrica 21, 608-610 (1953), dated erroneously 1963 by the authors] proved that every oriented graph in which every pair of vertices are joined by at most one edge can be realized as a graph associated with some R-matrix in this manner. Denote by $m(n)$ the smallest number such that every graph on n vertices corresponds to some $m \times n$ R-matrix. The main object of this paper is to obtain estimates for $m(n)$. *R.Stearns* (Zbl 090.25101) proved that $m(n) > c_2 n / \log n$, the authors prove that $m(n) \leq c_1 n / \log n$ (where c_1, c_2 are fixed positive constants). The paper is concluded with a number of unsolved problems.

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