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On complete topological subgraphs of certain graphs (In English)

Ann. Univ. Sci. Budapest. Rolando Eötvös, Sect. Math. 7, 143-149 (1964).

Let $G(n,l)$ a graph of n vertices and l edges. We say that $G(n,l)$ contains a complete k-gon if there are k vertices of $G(n,l)$ any two of which are connected by an edge, we say that it contains a complete topological k-gon if it contains k vertices any two of which are connected by paths no two of which have a common vertex (except of endpoints). Denote the complete k-gon by $\langle k\rangle$, and the complete topological k-gon by $\langle k\rangle$, and the complete topological k-gon by $\langle k\rangle_t$. The first theorem of the paper is the following: If $r\geq 2$ and $c_3\geq 1/(2r+2)$, then $G(n,c_3n^2)$ contains $\langle [c_4n^{1/r}]\rangle_t$, where c_4 depends on c_3. Denote by $f(k,l)$ the smallest integer such that splitting the edges of an $\langle f(k,l)\rangle$ into two classes in an arbitrary way, either the first contains a $\langle k\rangle$ or the second an $\langle l\rangle$. There are estimation for $f(k,l)$. [*P. Erdős* and *G. Szekeres*, Zbl 012.27010; *C. Frasney*, C. R. Acad. Sci., Paris 256, 2507-2510 (1963; Zbl 211.02502); *P. Erdős*, Zbl 032.19203; Zbl 097.39102) Similarly, denote by $f(k_t,l_t)$ the smallest integer such that splitting the edges of an $\langle f(k_tl_t)\rangle$ into two classes in an arbitrary way, either the first contains a $\langle k\rangle_t$ or the second an $\langle l\rangle_t$. Moreover $f(k,l_t)$ and $f(k_t,l)$ have a self-explanatory meaning. Theorem 2. gives the estimations

$$c_1k^2 < f(k_tl_t) < c_2k^2,$$

$$c_5l^{4/3}(\log l)^{-3/2} < f(3,l_t) \leq \binom{l+1}{2},$$

$$c_6k^3(\log k)^{-1} < f(k,k_t).$$

The symbol $m\to\infty(m_t,m_t)^2$ denotes the statement that if we split the edges of a complete graph of power m into two classes in an arbitrary way, then there exists a complete topological subgraph of power m all whose edges belong to the same class. The third theorem states $m\to(m_t,m_t)^2$ if m is an arbitrary cardinal. The paper contains further interesting results as collararies to the main theorems, and unsolved problems.

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Classification:
05C10 Topological graph theory