Zbl 105.17504

Erdős, Pál

Über ein Extremalproblem in der Graphentheorie.

On an extremal problem in graph theory. (In German)

Arch. Math. 13, Festschrift Reinhold Baer 222-227 (1962). [0003-889X]

The author considers graphs without loops or multiple edges. He proves that, if $n \geq 400k^2$, every graph with n vertices and at least

$$l = [\frac{1}{4}(n-k+1)^2] + (k-1)(n - \frac{1}{2}k + 1)$$

edges contains k disjoint triangles, with the exception of an (up to isomorphism) unique graph with n vertices and l edges. He states that a more complicated version of the proof can replace $400k^2$ by Ck, where C is an absolute constant. The following further theorems are stated with brief hints for proof. Let G be a graph with n vertices. (1) If $k \equiv n \pmod 2$ and every vertex has valency $\geq \frac{1}{2}(n+k)$ and $n \geq$ some $n_0(k)$, then G contains k disjoint triangles. (2) If $n > ck$, where c is a sufficiently large absolute constant, and G has $\frac{1}{4}n^2 + k$ edges, then G contains k triangles no two of which have a common edge.

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Classification:
05C35 Extremal problems (graph theory)