Zbl 101.11204

Erdős, Pál; Taylor, S.J.

On the Hausdorff measure of Brownian paths in the plane (In English)

Proc. Camb. Philos. Soc. 57, 209-222 (1961).

Let us denote by Ω the set of all brownian plane paths $z(t,\omega)=(z(t,\omega),y(t,\omega))$ where ω is a random point and $0< t<\infty$. One of the two authors (S.J. Taylor, Zbl 050.05803) constructed a probabilistical device $\{\Omega F,\mu\}$ for the space of Brownian motion.

Paul Lévy has proved (Zbl 024.13906) that the Lebesgue plane measure of the set $L(0,\infty;\omega)$ [where $L(a,b;\omega)=\{z(t,\omega)\mid 0\leq a< t< l\leq \infty\}$] is – with probability one – equal to null. In the present paper the authors prove Lévy's conjecture i. e. that, in contrast to the occurrences in the multidimensional case, the measure of the set $L(0,1;\omega)$ in the two dimensional space is finite, with respect to function $-x^2\log x$. The method employed in the demonstration uses the connexion between the Hausdorff-measure and the generalized capacity, that was pointed out by S.Kametani [Jap. J. Math. 19, 217-257 (1946; Zbl 061.22704)].

 $O.\,Onicescu$

Classification: 60J65 Brownian motion 28A78 Hausdorff measures