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Erdős, Pál; Ko, Chao; Rado, R.

Intersection theorems for systems of finite sets. (In English)

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Let $\{a_1, ..., a_2\}$ be a system of subsets of a set of finite cardinality m such that $a_\mu \not\subset a_\nu$ for $\mu \neq \nu$. The authors impose an upper limitation l on the cardinals of the sets a_ν, in symbols $|a_\nu| \leq l$, and a lower limitation k on the cardinals of the intersection of any two sets a_μ and a_ν, in symbols $|a_\mu a_\nu| \geq k$, and deduce upper estimates for the number n. If $k = 1$ and $1 \leq l \leq \frac{1}{2}m$, then $n \leq \binom{m-1}{l-1}$, the inequality being strict in case $|a_\nu| < l$ for some ν. Let $k \leq l \leq m$, $n \geq 2$ and either $2l \leq 1 + m$ or $2l \leq k + m$, $|a_\nu| = l$ for each ν. Then (i) either $|a_1 \cdots a_n| \geq k$, $n \leq \binom{m-k}{l-1}$ or $|a_1 \cdots a_n| < k < l < m$, $n \leq \binom{m-k-1}{l-k-1}\binom{l}{k}^3$; (ii) if $m \geq k+(l-k)\binom{l}{k}^3$, then $n \leq \binom{m-k}{l-k}$. Finally, the authors discuss the inequality imposed on m in (ii) and present some problems due to the replacement of the condition $a_\mu \not\subset a_\nu$ by $a_\mu \neq a_\nu$.

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05D05 Extremal set theory