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On a theorem in the theory of relations and a solution of a problem of Knaster. (In English)

Colloq. Math. 8, 19-21 (1961). [0010-1354]

Let S be a non void set; let $f : S \to PS$ be a mapping such that for every point $x \in S$ fx be a non-void subset of S; if $x, y \in S$ and $x \notin fy$, $y \notin fx$ the points x, y are said independent. A subset X of S is independent in respect to f provided every pair of distinct points of S are independent.
Theorem: If $S = I\omega_m$ and if there is some ordinal $\beta < \omega_m$ such that for every $x \in S$ the set fx is of an ordinal type $< \beta$, then there exists an independent subset of S of cardinality kS. (The theorem is a generalization of a similar statement in which a uniform boundedness of the cardinals kfx $(x \in S)$ intervenes.) The theorem is used to prove a conjecture of Knaster stating that there is no partition $A \cup B = R^2$ of plane R^2 such that every x-line would intersect the set A at a subset of ordinal number $< \beta$ and that every y-line would intersect B at a subset of ordinality $< \beta$; here one supposes give a normal well-order of the real continuum R inducing a determed well-order of any subset of R.

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Classification:
04A05 Relations, functions