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Erdős, Pál; Fodor, G.; Hajnal, András
On the structure of inner set mappings. (In English)
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Let I_1, I_2 be an ordered pair of sets and $G : I_1 \to I_2$ a mapping of I_1 into I_2; G is called an inner set mapping provided $GX \subset X$ for every $X \in I_1$. The inverse of any $X_0 \in I_2$ is defined in two ways: as $X_0^{-1} = \bigcup X$ $(GX = X_0)$ and as $X_0^{*-1} = \{X; G(X) = X_0\}$. For any set S and any cardinal n let $[S]^n$ and $[S]^{<n}$ denote the family of all subsets of S, each of the cardinality n and $< n$ respectively. The mappings on (resp. into) $[S]^n, [S]^{<n}$ are called of type (resp. of range) n and $< n$ respectively. For any cardinal n let n^* be the smallest cardinal such that n be the sum of n^* cardinals $< n$. In connection with set mappings of type q, and of range p of subsets of S, S being of cardinality m, let $((m, p, q)) \to r$ and $((m, p, q))^* \to r$ respectively mean that for every set mapping of $[S]^q$ into $[S]^p$ there exists an $X_0 \in [S]^p$ satisfying card $X_0^{-1} = r$ and card $X_0^{*-1} = r$ respectively. Analogously the authors define $((m, < p, q))^* \to r$. Twelve theorems and several problems concerning the foregoing notions are proved and formulated respectively; here are some ones.
Theorem 3: $q \geq \aleph_0 \Rightarrow ((m, q, q)) \not\to q^+$.
Theorem 5: $q \geq \aleph_0 \Rightarrow ((m, q, q))^* \not\to 2$. Theorem 6: $p < q$, $q^p < m^q$, $q \geq \aleph_0$, $q^p < m^* \Rightarrow ((m, p, q)) \to m$. If moreover the general continuum hypothesis is assumed and $q^p \neq m^*$ or $q \geq m^*$, then $((m, p, q))^* \to m^q$ and $((m, p, q)) \to m$ (Theorem 9). Let α be an ordinal; if $0 < k < l < \aleph_0$, then $((\aleph_{\alpha+k}, k, l)) \to \aleph_\alpha$ (Theorem 10) but $((\aleph_{\alpha+k}, k, l)) \not\to \aleph_{\alpha+1}$ (Theorem 11). If q is infinite and regular and $r^n < m$ for every $r < q$ and $n < q$ then $((m, < q, q)) \to m$ (Theorem 12). Problems: Does subsist $((\aleph_{\omega_{\omega+1}}, \aleph_0, \aleph_\omega)) \to \aleph_{\omega_{\omega+1}}$? If $n > \aleph_\omega$, does $((m, < \aleph_\omega, \aleph_\omega)) \to n$ hold for some m?

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Classification:
05D10 Ramsey theory
04A20 Combinatorial set theory
03E05 Combinatorial set theory (logic)

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