Zbl 074.03502
Bateman, Paul T.; Erdős, Pál
Monotonicity of partition functions. (In English)
Mathematika, London 3, 1-14 (1956).

Let A be an arbitrary set of different positive integers (finite or infinite) other than the empty set or the set consisting of the single element unity. Let $p(n) = p_A(n)$ denote the number of partitions of the integer n into parts taken from the set A, repetitions being allowed. Let k be any integer and suppose we define $p^{(k)}(n) = p_A^{(k)}(n)$ by the formal power- series relation

$$f_k(X) = \sum_{n=0}^{\infty} p^{(k)}(n)X^n = (1-X)^k \sum_{n=0}^{\infty} p(n)X^n =$$

$$= (1-X)^k \prod_{a\in A} (1-X^a)^{-1}.$$

For $k \geq 0$, the authors prove that $p^{(k)}(n)$ is positive for all sufficiently large positive integers n if and only if A has the property P_k, viz. there are more than k elements in A and, if we remove an arbitrary subset of k elements from A, the remaining elements have greatest common divisor unity. Among a number of other results we may select as typical: For any k, let A infinite and have property P_k and let $n \to \infty$. Then $p^{(k)}(n)n^{-c} \to +\infty$ for any fixed c. Also, if

$$\varrho^{(k)}(n) = p^{(k+1)}(n)/p^{(k)}(n) = 1 - p^{(k)}(n-1)/p^{(k)} \mid (n),$$

then $\varrho^{(k)}(n) \to 0$, $n\varrho^{(k)}(n)$ is unbounded above and $n\varrho^{(k-1)}(n) \to +\infty$.

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Classification:
11P82 Analytic theory of partitions