

Zbl 070.04104

Erdős, Pál; Fuchs, W.H.J.

On a problem of additive number theory. (In English)

J. London Math. Soc. 31, 67-73 (1956).

Let $\{a_i\}$ be a non decreasing infinite sequence of non negative integers, $f(n)$ the number of solutions of $a_i + a_j = n$ and $r(n)$ the number of solution of $a_i + a_j \leq n$. Erdős and Turán [J. London Math. Soc. 16, 212-215 (1941; Zbl 061.07301)] conjectured that $r(n) - cn = O(1)$ cannot hold. In the present paper the authors prove (1) If $c > 0$, then $r(n) = cn + o(n^{1/4} \log^{-1/2} n)$ cannot hold. (2) If $c > 0$, or $c = 0$ and $a_k < Ak^2$ then

$$\overline{\lim}_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^n (f(k) - c)^2 > 0.$$

Theorem 2. contains a theorem of Dirac-Newman (Zbl 043.04702) who proved that $f(n)$ cannot be a constant for $n \geq n_0$.

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Classification:

11B34 Representation functions