Zbl 056.03505
Ankeny, N.C.; Erdős, Pál
The insolubility of classes of diophantine equations. (In English)
Amer. J. Math. 76, 488-496 (1954).

Let m be a natural and $a_1, a_2, ..., a_n$ non-zero rational integers such that for every selection $e_j = 0$ or ± 1 $(j = 1, 2, ..., n)$ except $e_1 = e_2 = \cdots = e_n = 0$ we have $a_1 e_1 + \cdots + a_n e_n \neq 0$. Let U be a large positive real number tending to infinity and $D(U)$ the number of $m \leq U$ for which the equation $a_1 X_1^m + a_2 X_2^m + \cdots + a_n X_n^m = 0$ has rational integer solutions in the variables $X_1, X_2, ..., X_n$ where not all $X_j = 0$. The authors prove that $D(U) = o(U)$. They also prove a case which is excluded in the theorem above: The density of integers m, for which the equations $X_1^m + X_2^m + X_3^m = 0$ has a rational solution and for which $(X_1 X_2 X_3, m) = 1$, is zero. They also mention that the first result can be generalized from the rational number field to any algebraic number field F. The paper contains various misprints and defects. Lemma 4 is incorrect.

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Classification:
11D41 Higher degree diophantine equations
14G05 Rationality questions, rational points