Zbl 025.18701
Erdős, Paul
The dimension of the rational points in Hilbert space. (In English)
Ann. of Math., II. Ser. 41, 734-736 (1940).

Let H denote the Hilbert space of all sequences of real numbers $(x_1, x_2, ...)$ such that $\sum_{i=1}^{\infty} x_i^2 < \infty$. Let R be the set of points of H having all coordinates rational. Let R_0 be the set of points of H of the form $\left(\frac{1}{n_1}, \frac{1}{n_2}, \cdots\right)$, where the n_i are positive integers. Let R_1 be the closure of R_0. The author shows that R_0, R_1 and R have the dimension 1.

As the cartesian product $R_1 \times R_1$ is homeomorphic to R_1, it follows that there exists a metric separable complete space X such that X and $X \times X$ have dimension 1.

Béla de.Sz.Nagy (Szeged)

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