Zbl 015.10001
Davenport, H.; Erdős, Pál
On sequences of positive integers. (In English)
Acta Arith. 2, 147-151 (1936). [0065-1036]

Let $a_1, a_2, ...$ be any sequence of different positive integers, and $b_1, b_2, ...$ the integers divisible by at least on a. It was proved by *A.S.Besicovitch* (Zbl 009.39504) that the sequence $\{b_i\}$ may have different upper and lower densities. Here it is shown that the logarithmic density

$$\lim_{x \to \infty} (\log x)^{-1} \sum_{b_i \leq x} b_i^{-1}$$

exists and is equal to the lower density of the sequence. The proof uses Dirichlet series. It is deduced that if a sequence $a_1, a_2, ...$ has a positive upper logarithmic density, then it has a subsequence $a_{i_1}, a_{i_2}, ...$ in which $a_{i_k} \mid a_{i_{k+1}}$ $(k = 1, 2, ...)$.

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Classification:
11B83 Special sequences of integers and polynomials
11B05 Topology etc. of sets of numbers